

On the role of resonance absorption in flows receptive to the magnetorotational instability

Mattias Brynjell-Rahkola¹, Gordon I. Ogilvie¹

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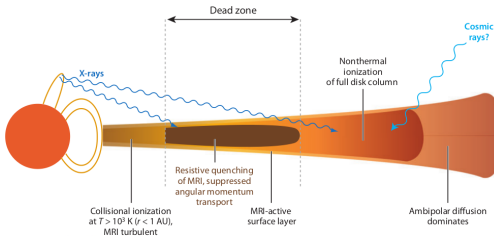
UK & Ireland Discs Meeting 2024
11 September 2024, University of Warwick (UK)



Engineering and
Physical Sciences
Research Council

Sustenance of turbulence/dynamo action in protoplanetary discs

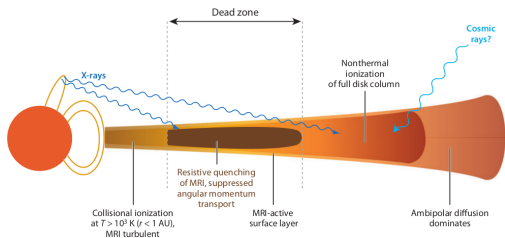
- Presence of a linear instability mechanism?
 - Degree of ionization.
 - Different non-ideal MHD terms.
 - ...



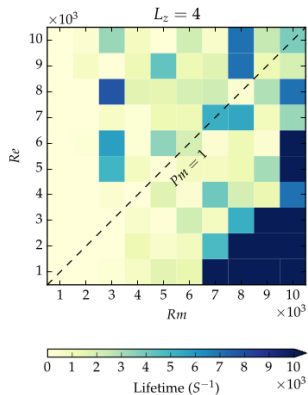
Armitage (2011).

Sustenance of turbulence/dynamo action in protoplanetary discs

- Presence of a linear instability mechanism?
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 - ...
- Subcritical turbulence:
 - Laminar and turbulent state in competition.**

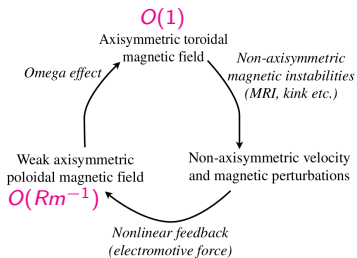


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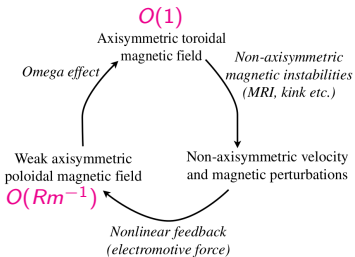
Nauman & Pessah (2018).

Self-sustaining processes in magnetohydrodynamics (MHD)

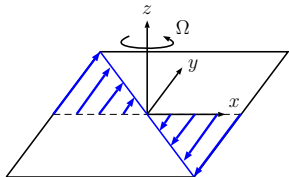


Riols et al. (2013).

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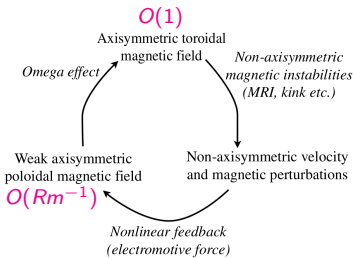


Field decomposition:

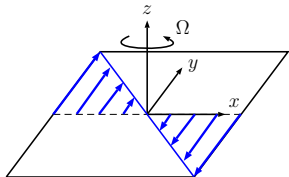
$$\mathbf{v} = \underbrace{\bar{v}_y \mathbf{e}_y + \bar{\mathbf{v}}_p}_{y\text{-averaged part}} + \underbrace{\mathbf{v}_{3D}}_{\text{Wave}}, \quad \mathbf{b} = \underbrace{\bar{b}_y \mathbf{e}_y + \bar{\mathbf{b}}_p}_{y\text{-averaged part}} + \underbrace{\mathbf{b}_{3D}}_{\text{Wave}}$$

$$\bar{\mathbf{v}}_p = \nabla \times (\psi \mathbf{e}_y), \quad \bar{\mathbf{b}}_p = \nabla \times (\chi \mathbf{e}_y)$$

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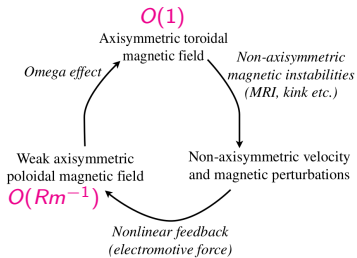
Toroidal field:

$$\frac{\partial \bar{b}_y}{\partial t} = \underbrace{-\bar{b}_x}_1 + \mathbf{e}_y \cdot \overline{\nabla \times (\mathbf{v} \times \mathbf{b})} + \frac{1}{Rm} \nabla^2 \bar{b}_y$$

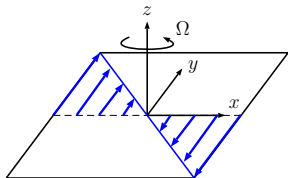
Poloidal flux:

$$\frac{\partial \chi}{\partial t} + \frac{\partial (\psi, \chi)}{\partial (z, y)} = \underbrace{(\mathbf{v}_{3D} \times \mathbf{b}_{3D}) \cdot \mathbf{e}_y}_2 + \frac{1}{Rm} \nabla^2 \chi$$

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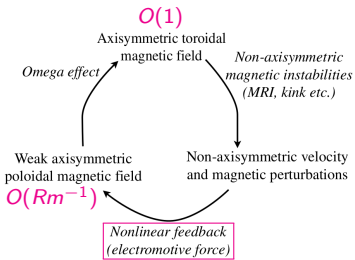
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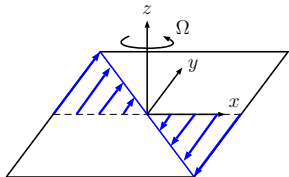
- ① Ω -effect
- ② Feedback electromotive force (EMF)

(Rincon et al. 2007, Rincon et al. 2008)

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Research question:

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Consider a system with a continuous background flow $\mathbf{V}(\mathbf{x})$, a magnetic field $\mathbf{B}(\mathbf{x})$, and a wave with wavenumber vector \mathbf{k} giving the flow frequency $\omega_f = \mathbf{V} \cdot \mathbf{k}$ and Alfvén frequency $\omega_A = \mathbf{B} \cdot \mathbf{k}$:

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- At these resonance points, energy is **absorbed** due to **phase mixing**.
- This absorption process is **independent** on **resistivity** and **viscosity**.
- As a consequence of the absorption, resonance layers may act as **barriers** that delimit fluid/plasma motion.

(Barston, 1964; Booker & Bretherton, 1967; Grad, 1969; Uberoi, 1972)

Analytical solution near resonance

- Consider the linearized **incompressible ideal Euler** and **induction equation** in **cylindrical coordinates**

$$\partial_t \mathbf{v} + (\mathbf{V} \cdot \nabla) \mathbf{v} + (\mathbf{v} \cdot \nabla) \mathbf{V} - (\mathbf{B} \cdot \nabla) \mathbf{b} - (\mathbf{b} \cdot \nabla) \mathbf{B} = -\nabla \pi$$

$$\partial_t \mathbf{b} + (\mathbf{V} \cdot \nabla) \mathbf{b} + (\mathbf{v} \cdot \nabla) \mathbf{B} - (\mathbf{B} \cdot \nabla) \mathbf{v} - (\mathbf{b} \cdot \nabla) \mathbf{V} = 0$$

$$\nabla \cdot \mathbf{v} = 0, \quad \nabla \cdot \mathbf{b} = 0,$$

with a basic state:

$$\mathbf{V}(r) = [0, V_\theta(r), V_z(r)], \quad \mathbf{B}(r) = [0, B_\theta(r), B_z(r)],$$

and a perturbation $[\mathbf{v}, \mathbf{b}, \pi](r, \theta, z, t) = [\hat{\mathbf{v}}, \hat{\mathbf{b}}, \hat{\pi}](r) e^{i(m\theta + kz - \omega t)}$.

(Goossens *et al.* 1992)

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- By solving the induction equation for $\hat{\mathbf{b}}$ and expressing $\hat{\mathbf{v}}$ in terms of a Lagrangian displacement ξ , one obtains the equation system

$$\begin{bmatrix} \left(C_1 - \frac{D}{r}\right) & -C_2 \\ C_3 & -C_1 \end{bmatrix} \begin{pmatrix} \xi_r \\ \pi \end{pmatrix} = D \begin{pmatrix} \xi_r' \\ \pi' \end{pmatrix},$$

where C_1 , C_2 and C_3 are coefficients that depend on m , k , r , \mathbf{B} , \mathbf{V} and

$$\omega_f = \frac{V_\theta}{r} m + V_z k, \quad \Omega = \omega - \omega_f, \quad \omega_A = \frac{B_\theta}{r} m + B_z k, \quad D = \Omega^2 - \omega_A^2.$$

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Analytical solution near resonance cont.

- The system has a singularity at $r = r_A$ where $D = 0$. Let $s = r - r_A \ll 1$ and assume that ξ_r and π may be described by modified power series solutions in s :

$$\xi_r(s) = \sum_{n=0}^{\infty} X_n s^{\sigma+n}, \quad \pi(s) = \sum_{n=0}^{\infty} Y_n s^{\sigma+n}$$

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Analytical solution near resonance cont.

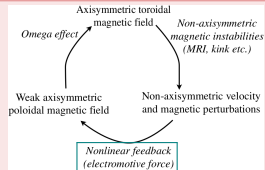
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- Taylor expand all functions of ω in $\alpha \ll 1$.

Electromotive force (EMF)

$$\text{Recall: } \partial_t \chi = (\mathbf{v}_{3D} \times \mathbf{b}_{3D}) \cdot \mathbf{e}_y + \dots$$



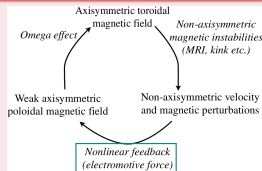
Electromotive force (EMF)

- Describes the induction of the magnetic field.
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- Study the **steady mean** component of the forcing.

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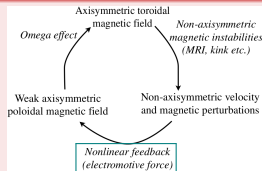
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Substitution of the analytical solution:

$$r: A e^{\pm \phi \sigma_i} \left\{ \frac{2kT}{D(1)(m^2 + k^2 r_A^2)} \frac{1}{s^2 + \chi^2} \times \right. \\ \left[sr_A \left(\Omega \omega'_A - \Omega' \omega_A \right) \mp \chi \left(kr_A^2 \left(\omega_f \left(\frac{V_\theta}{r} \right)' + \Omega \left(\frac{B_\theta}{r} \right)' \right) - m \left(\omega_f V'_z + \Omega B'_z \right) \right) \pm \alpha \omega_f \frac{4kT}{D(1)} - \right. \\ \left. \left. \alpha \chi r_A \omega'_f \mp \alpha s \left(\left(\frac{B_\theta}{r} \right)' kr_A^2 - B'_z m \right) \right] + r_A \left(B'_z \left(\frac{V_\theta}{r} \right)' - V'_z \left(\frac{B_\theta}{r} \right)' \right) \right\}, \\ \theta: 4A\alpha \frac{kT\omega_f}{D(1)(m^2 + k^2 r_A^2)} e^{\pm \phi \sigma_i} \frac{kr_A s \mp m\chi}{s^2 + \chi^2}, \quad z: 4A\alpha \frac{kT\omega_f}{D(1)(m^2 + k^2 r_A^2)} e^{\pm \phi \sigma_i} \frac{ms \pm kr_A \chi}{s^2 + \chi^2},$$

Validation of the EMF expression

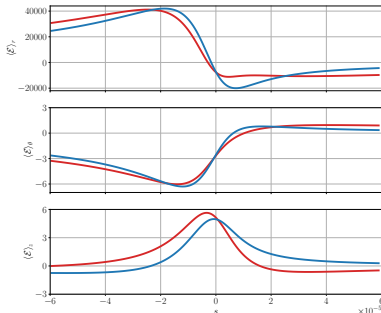
Numerical ideal MHD calculations (Ogilvie & Pringle, 1996):

Background state: $\mathbf{V}(r) = [0, r\Omega_K(r), 0]$, $\mathbf{B}(r) = [0, B_0 r^{-1}, 0]$,
with $B_0 = 0.2$ and $\Omega_K(r) = r^{-3/2}$ is the Keplerian profile.

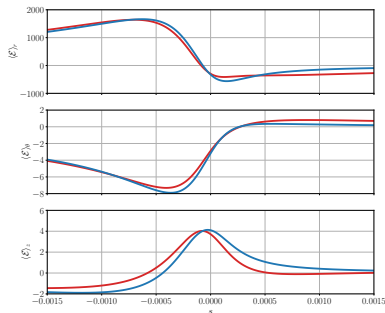
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$$k = 2, m = 5, \omega = 4.00 + i5.94 \times 10^{-5}.$$

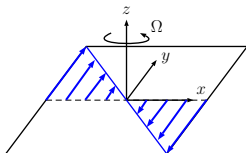


$$k = 3, m = 5, \omega = 4.00 + i1.66 \times 10^{-3}.$$

Numerical, Analytical

Cartesian model: stability diagram

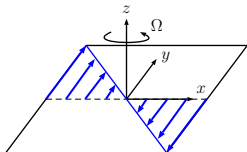
No-slip, electr. conducting walls; $Re = 10^3$, $Pm = 1$, $R = -4/3$; $\mathbf{B} = (0, B_0, 0)$



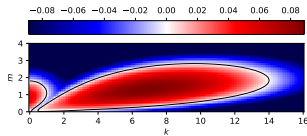
Rotating plane Couette flow

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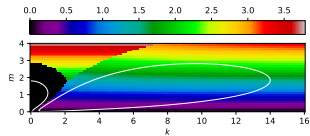
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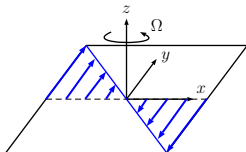
Growth rates



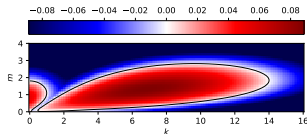
Frequencies

Cartesian model: stability diagram

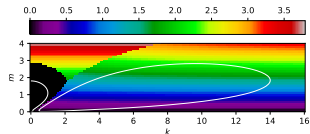
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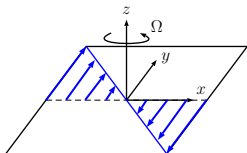


Frequencies

$$k = 4, m = 2,$$
$$\omega = 1.49 - i0.00$$

Cartesian model: stability diagram

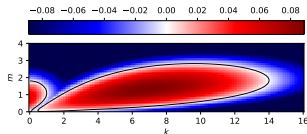
No-slip, electr. conducting walls; $Re = 10^3$, $Pm = 1$, $R = -4/3$; $\mathbf{B} = (0, B_0, 0)$



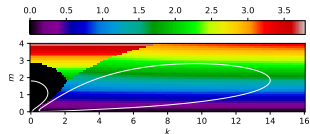
Rotating plane Couette flow

$$k = 4, m = 2,$$

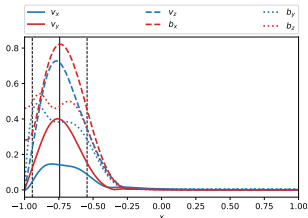
$$\omega = 1.49 - i0.00$$



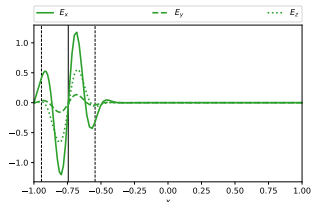
Growth rates



Frequencies



Eigenfunction



EMF

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Thank you for your attention!