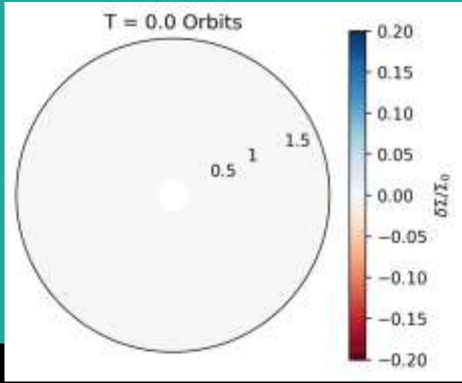


$$\frac{M_p}{M_{th}} = 0.25$$
$$h_p = 0.05$$
$$\Sigma_0 \propto R^{-3/2}$$

How do planets carve smooth gaps in inviscid discs? (Analytically)

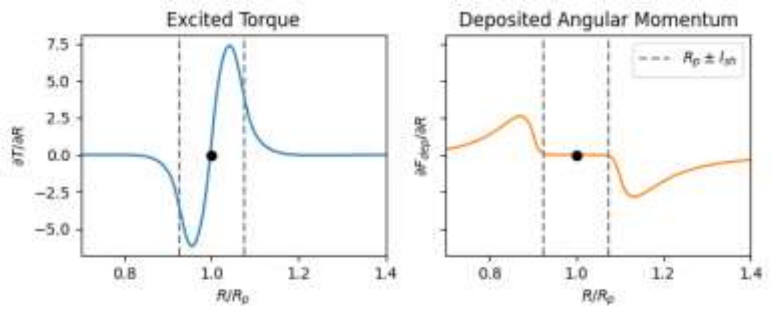
Amelia Cordwell, DAMTP, Cambridge ajc356@cam.ac.uk
Cordwell & Rafikov (2024)

Existing theory

Planets excite waves, these waves then dissipate angular momentum into the disc. In adiabatic discs this happens primarily through shock formation (Rafikov 2002a, Miranda & Rafikov 2020).

Viscous steady state of a planet disc interaction has been given detailed attention in literature (e.g. Kanagawa et al. 2015, Duffel et al. 2020)

What's missing? Time dependence and low viscosity discs!

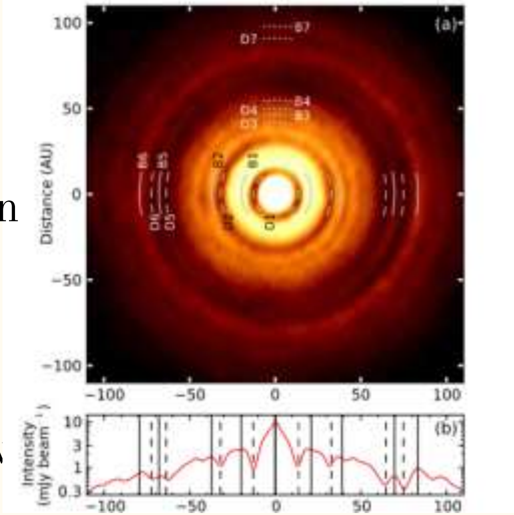


$M_p = 0.25 M_{th}, h_p = 0.05, p = 1.5, t = 100 P_p$

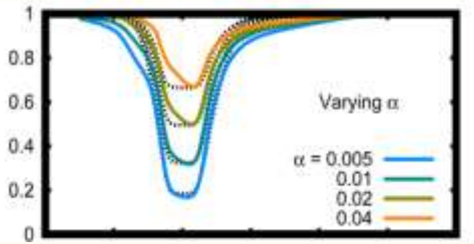
$$\frac{\Sigma_p}{\Sigma_0} = \frac{1}{1 + 0.04\,K},$$

$$K = \left(\frac{M_p}{M_*}\right)^2 h_p^{-5} \alpha^{-1}$$

Kanagawa et al. 2015 (a, b)



ALMA Partnership et al. 2015



Duffel et al. 2020

Naive time dependence **doesn't work**

Consider the evolution of an inviscid disc

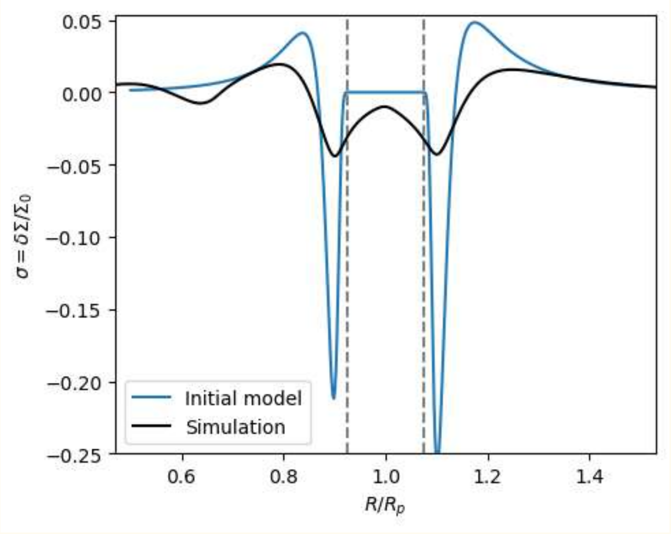
In early stages of gap formation $\Omega \approx \Omega_K$ so following e.g. Pringle 1981, Rafikov 2002b, we assume $\partial l/\partial t \approx 0$, hence $l \equiv R^2\Omega$

$$\frac{\partial \Sigma}{\partial t} = \frac{-1}{R}\frac{\partial}{\partial R}\left[\left(\frac{\partial l}{\partial R}\right)^{-1}\left\{\frac{1}{2\pi}\frac{\partial F}{\partial R}\right\}\right]$$

Take the known model for $\partial F/\partial R$ from either simulations or Cimerman & Rafikov (2023)

$$R\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}(R\Sigma v_R) = 0$$

$$R\frac{\partial}{\partial t}(\Sigma R^2\Omega) + \frac{\partial}{\partial R}(R\Sigma v_R \Omega R^2) = \frac{1}{2\pi}\frac{\partial F_{dep}}{\partial R}$$



$M_p = 0.25M_{th}, h_p = 0.05, p = 1.5, t = 100P_p$

$$\Omega \approx \Omega_k \text{ does not imply } \frac{\partial l}{\partial t} = 0$$

$$\frac{\partial \Sigma}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left[\left(\frac{\partial l}{\partial R}\right)^{-1}\left\{\frac{-1}{2\pi}\frac{\partial F}{\partial R} + \Sigma R \frac{\partial l}{\partial t}\right\}\right]$$

That sounds hard to include :(**But we can do it!**

Model specific angular momentum by $\Omega^2 = \Omega_K^2 + \frac{1}{R\Sigma}\frac{\partial P}{\partial R}$ and $P = c_s^2 \Sigma$

Then for a shallow gap in a we can derive the solution

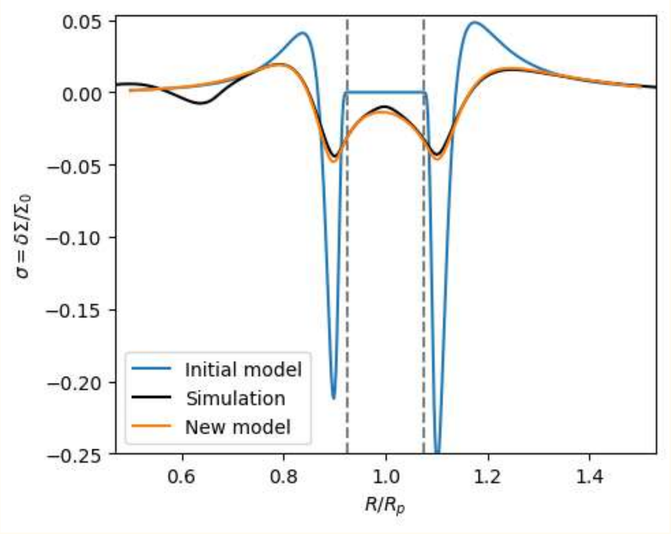
$$\frac{\delta\Sigma}{\Sigma_0}(R,t) = 2\,t\,R^{(p-3)/2}\,\frac{R_p}{h_p^2} \times \left[K_{3-p}\left(y(R)\right)\int_R^\infty I_{3-p}\left(y(x)\right)x^{(p-3)/2}S(x)dx - I_{3-p}\left(y(R)\right)\int_0^R K_{3-p}\left(y(x)\right)x^{(p-3)/2}S(x)dx\right]$$

where

$$h_p \equiv \frac{c_s}{\Omega_K R_p} \qquad \Sigma_0 \propto R^{-p}$$

$$S(R) \equiv \frac{1}{\pi}\frac{\partial}{\partial R}\left[\frac{R^{-p-1}}{\Omega_K}\Sigma_0^{-1}\frac{\partial F_{dep}}{\partial R}\right]$$ and $$y(x) = \frac{2}{h_p}\left(\frac{x}{R_p}\right)^{-1/2}$$

Which, despite its confusing appearance, represents a convolution over the angular momentum deposition function



$M_p = 0.25 M_{th}, h_p = 0.05, p = 1.5, t = 100 P_p$

(An implementation of this is available on github! https://github.com/cordwella/vortensity_evolution)

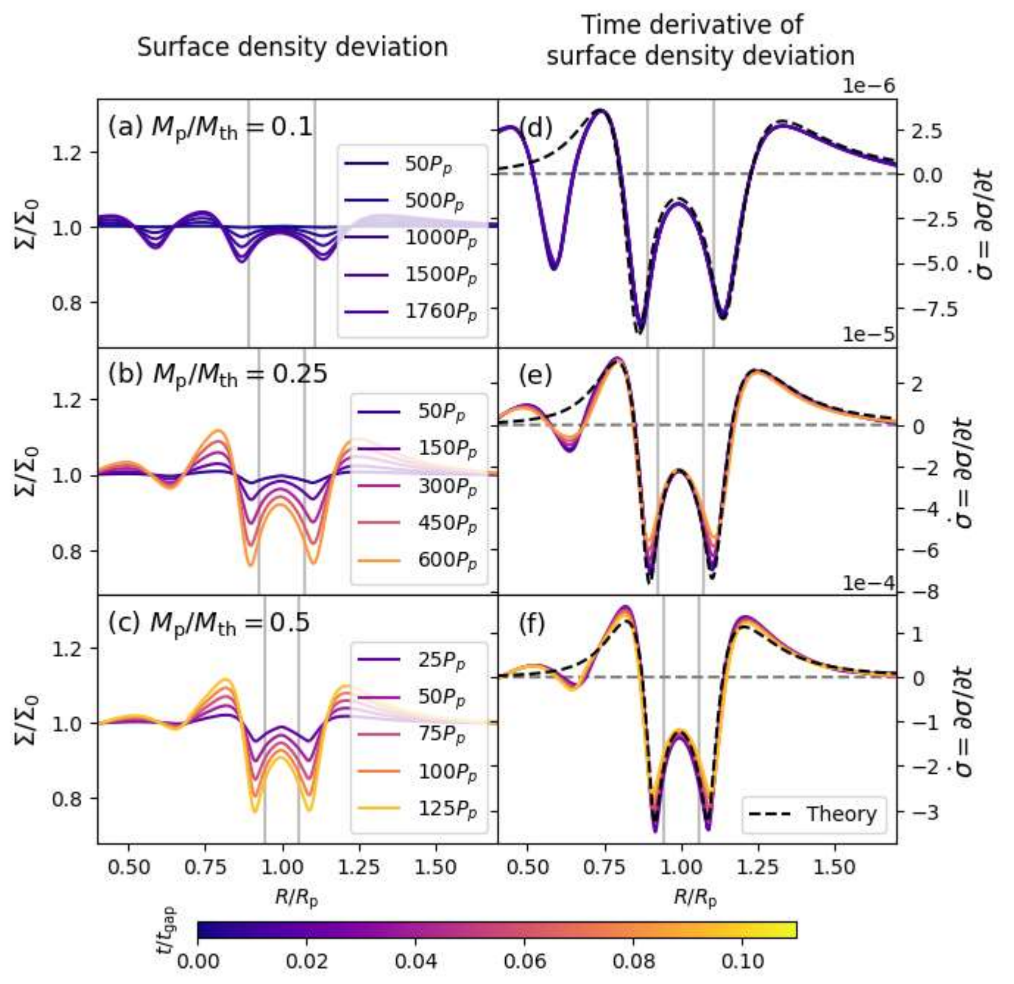
Does this work generally?

Different masses? Yep

Viscous discs? For a period of time $t_{l-\nu} \approx \frac{P_p}{6\pi\alpha}$

Evolving planets? The theory is easily extendable

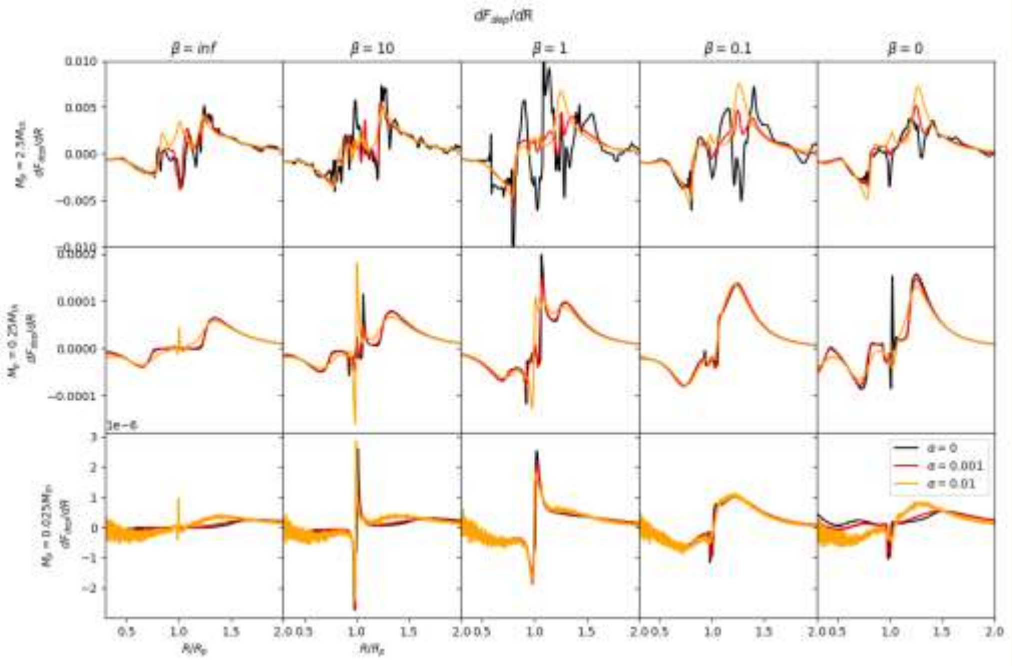
Deep gaps? Restricted to 20% gas gaps - but this can imply observable dust rings/gaps



Wow! Amelia's theory works brilliantly, time to give her lots of money, a PhD and every possible award. We should pack up the entire field and go home.

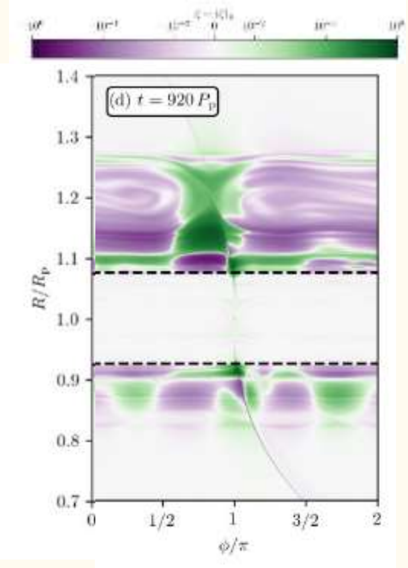
Nope! Thermodynamics and non-linear effects in deep gaps

Thermodynamics can significantly affect angular momentum deposition (forthcoming work)

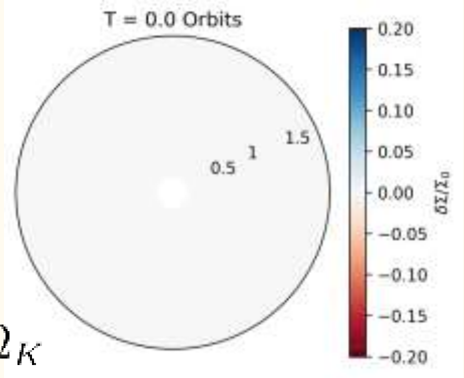


F_{dep} can be strongly impacted by the structure of deep gaps

Once a gap gets deep enough, the RWI causes the formation of vortices (e.g. Cimerman & Rafikov 2023)



Conclusion



1. You should care about time dependence in gaps
2. The time dependence of Omega is important even when $\Omega \approx \Omega_K$
3. Even in inviscid discs, gaps are smooth - pressure acts as a smoothing force
4. In inviscid (adiabatic) gaps there is a characteristic double gap shape
   a. This shape is seen in observations: e.g. HL Tauri, AS209
5. The linear model is easy to use and works well
   a. See: https://github.com/cordwella/vortensity_evolution for an implementation
6. However to make further progress we need to understand angular momentum deposition better - especially with different thermodynamics

For a full description see Cordwell & Rafikov (2024), MNRAS